C.U.SHAH UNIVERSITY
Summer Examination-2018

Subject Name : Engineering Mathematics – I

Subject Code : 4TE01EMT3   **Branch: B.Tech (All)**

Semester : 1   **Date : 21/03/2018**   **Time : 02:30 To 05:30**   **Marks : 70**

Instructions:
- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

a) nth derivative of $y = \dfrac{1}{x+a}$ is

(A) $\dfrac{(-1)^n \, n!}{(x+a)^{n+1}}$ (B) $\dfrac{(-1)^{n-1} \, n!}{(x+a)^{n+1}}$ (C) $\dfrac{(-1)^n \, n!}{(x+a)^n}$ (D) none of these

b) If $y = \cosh 2x$ and if n is even, then y_n equal to

(A) $2^n \sinh 2x$ (B) $2^n \cosh 2x$ (C) $\cosh(2nx)$ (D) none of these

c) If $y = \log(1+x)$, then x equal to

(A) $1 + y + \dfrac{y^2}{2!} + \dfrac{y^3}{3!}$ (B) $1 - \dfrac{y^2}{2!} + \dfrac{y^4}{4!} - .....$ (C) $y + \dfrac{y^2}{2!} + \dfrac{y^3}{3!} + \dfrac{y^4}{4!} + ...$

(D) none of these

d) If $y = \sin^{-1} x$, then x equal to

(A) $1 - \dfrac{y^2}{2!} + \dfrac{y^4}{4!} - .....$ (B) $1 + y + \dfrac{y^2}{2!} + \dfrac{y^3}{3!} + ...$ (C) $1 - y + \dfrac{y^2}{2!} - \dfrac{y^3}{3!} + ...$

(D) $y - \dfrac{y^3}{3!} + \dfrac{y^5}{5!} - .....$

e) $\lim\limits_{x \to \infty} x \left(a^{\frac{1}{x}} - 1 \right) = $ _____

(A) $\log_e a$ (B) 0   (C) 1   (D) none of these

f) $\lim\limits_{x \to \infty} \dfrac{x^n}{e^{ax}} = $ _____

(A) -1 (B) 0   (C) 1   (D) none of these

g) If $x = r \cos\theta,\ y = r \sin\theta,$ then

(A) $\dfrac{\partial x}{\partial r} = \dfrac{\partial r}{\partial x}$ (B) $\dfrac{\partial x}{\partial \theta} = 0$ (C) $\dfrac{\partial x}{\partial r} = 0$ (D) $\dfrac{\partial x}{\partial r} = \dfrac{1}{\partial r / \partial x}$

h) If $f(x, y) = 0$, then $\dfrac{dy}{dx}$ is equal to

(A) $\dfrac{\partial f / \partial x}{\partial f / \partial y}$ (B) $\dfrac{\partial f / \partial y}{\partial f / \partial x}$ (C) $-\dfrac{\partial f / \partial y}{\partial f / \partial x}$ (D) $-\dfrac{\partial f / \partial x}{\partial f / \partial y}$

i) If $f_1 = \dfrac{vw}{u}, f_2 = \dfrac{wu}{v}, f_3 = \dfrac{uv}{w}$; then $\dfrac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)}$ is equal to

(A) 0 (B) 1 (C) 3 (D) none of these

j) $\dfrac{\partial(u, v, w)}{\partial(x, y, z)} \cdot \dfrac{\partial(x, y, z)}{\partial(u, v, w)} = $ _____

(A) 2 (B) 1 (C) 0 (D) none of these

k) If $x_r = \cos\left(\dfrac{\pi}{2^r}\right) + i\sin\left(\dfrac{\pi}{2^r}\right)$ then $x_1 x_2 x_3 \ldots$ to ∞ is

(A) -3 (B) -2 (C) -1 (D) 0

l) If $\left(\dfrac{1-i}{1+i}\right)^{100} = a + ib$, then

(A) $a = 2, b = -1$ (B) $a = 1, b = 0$ (C) $a = 0, b = 1$ (D) $a = -1, b = 2$

m) The system of equations $x + 2y + 3z = 1$, $x - y + 4z = 0$, $2x + y + 7z = 1$ has

(A) exactly one solution (B) only two solutions (C) no solution
(D) infinitely many solutions

n) The product of the eigenvalues of $\begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$ is

(A) 2 (B) 4 (C) 6 (D) 0

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** (14)

a) If $y = \dfrac{1}{x^2 + a^2}$ then find y_n. (5)

b) Expand $e^{\sin x}$ as a series of ascending power of x upto x^4. (5)

c) If $u = \tan^{-1}\left(\dfrac{y}{x}\right)$ then verify that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$. (4)

Q-3 **Attempt all questions** (14)

a) If $y = \sin\left(m \sin^{-1} x\right)$ then prove that (5)

$$\left(1 - x^2\right) y_{n+2} - \left(2n + 1\right) xy_{n+1} + \left(m^2 - n^2\right) y_n = 0$$

b) Prove that $\dfrac{x}{e^x - 1} = 1 - \dfrac{1}{2}x + \dfrac{1}{12}x^2 - \ldots$ (5)

c) Evaluate: $\lim\limits_{x \to a} \log\left(2 - \dfrac{x}{a}\right) \cot(x - a)$ (4)

Q-4 **Attempt all questions** (14)

a) Evaluate: $\lim\limits_{x \to 0} \dfrac{a}{x^2}\left[\dfrac{\sin kx}{\sin lx} - \dfrac{k}{l}\right]$ (5)

b) If $u = \dfrac{y^2}{x}$, $v = \dfrac{x^2}{y}$, evaluate $J = \left(\dfrac{x, y}{u, v} \right)$ and $J' = \left(\dfrac{u, v}{x, y} \right)$ and hence verify that $JJ' = 1$. **(5)**

c) Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem. **(4)**

Q-5   **Attempt all questions** **(14)**

a) If $u = \log\left(x^3 + y^3 - x^2 y - xy^2 \right)$ then prove that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} \right)^2 u = -\dfrac{4}{(x+y)^2}$. **(5)**

b) Evaluate: $\lim\limits_{x \to 0} \left(\dfrac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$ **(5)**

c) If $y = \sin^4 x$ then find y_n. **(4)**

Q-6   **Attempt all questions** **(14)**

a) The power consumed in an electric resistor is given by $P = \dfrac{E^2}{R}$ (in watts). If $E = 200$ volts and $R = 8$ ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0,20 ohms? **(5)**

b) Prove that $\left(\dfrac{1 + \sin\theta + i\cos\theta}{1 + \sin\theta - i\cos\theta} \right)^n = \cos\left(\dfrac{n\pi}{2} - n\theta \right) + i\sin\left(\dfrac{n\pi}{2} - n\theta \right)$ **(5)**

c) Find the rank of matrix $A = \begin{bmatrix} 6 & 1 & 1 & 1 \\ 16 & 1 & -1 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix}$. **(4)**

Q-7   **Attempt all questions** **(14)**

a) Using matrix method, show that the equations **(5)**
$3x + 3y + 2z = 1$, $x + 2y = 4$, $10y + 3z = -2$, $2x - 3y - z = 5$
Are consistent and hence find the solution.

b) Find the roots common to the equations $x^4 + 1 = 0$ and $x^6 - i = 0$. **(5)**

c) Find $\tanh x$ if $5\sinh x - \cosh x = 5$. **(4)**

Q-8   **Attempt all questions** **(14)**

a) Find the inverse of $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan reduction method. **(5)**

b) Using De Moivre's theorem, expand $\sin^8 \theta$ in a series of cosines of multiples of θ. **(5)**

c) Check whether the following set of vectors is linearly dependent or linearly independent: **(4)**
$(1, 2, -1, 0)$, $(1, 3, 1, 2)$, $(4, 2, 1, 0)$, $(6, 1, 0, 1)$